OKLAHOMA STATE UNIVERSITY

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING



ECEN 5713 Linear Systems Fall 2002 Final Exam



Choose any four out of five problems. *Please specify which four listed below to be graded:* 1)___; 2)__; 3)__; 4)__;

Name : ______

Student ID: _____

E-Mail Address:_____

Problem 1:

Consider the system

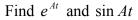
$$x(k+1) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 2 \\ 3 \end{bmatrix} u(k),$$
$$y(k) = \begin{bmatrix} 1 & 1 \end{bmatrix} x(k)$$

and let x(0) = 0 and $u(k) = 1, n \ge 0$.

- a) Determine $\{y(k)\}, k \ge 0$ by any approach.
- b) If it is known that when u(k) = 0, then y(0) = y(1) = 1, can x(0) be uniquely determined? If your answer is affirmative, determine x(0).

Problem 2: Let

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix},$$



Problem 3: Show that there exists a similarity transformation matrix P such that $\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \end{bmatrix}$

$$PAP^{-1} = A_{c} = \begin{vmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\alpha_{0} & -\alpha_{1} & -\alpha_{2} & \cdots & -\alpha_{n-1} \end{vmatrix},$$

if and only if there exists a vector $b \in \Re^n$ such that the rank of $\begin{bmatrix} b & Ab & \cdots & A^{n-1}b \end{bmatrix}$ is *n*.

<u>Problem 4</u>: Consider the matrix

$$A = \begin{bmatrix} -\alpha_1 & -\alpha_2 & -\alpha_3 & -\alpha_4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

show that its characteristic polynomial is given by

$$\Delta(\lambda) = \lambda^4 + \alpha_1 \lambda^3 + \alpha_2 \lambda^2 + \alpha_3 \lambda + \alpha_4.$$

Show also that if λ_i is an eigenvalue of *A*, then $\begin{bmatrix} \lambda_i^3 & \lambda_i^2 & \lambda_i & 1 \end{bmatrix}^T$ is an eigenvector pf A associated with λ_i .

<u>Problem 5</u>: Consider the system representations given by

$$x(k+1) = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(k)$$
$$y(k) = \begin{bmatrix} 1 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 1 & 0 \end{bmatrix} u(k)$$

and

$$\widetilde{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \widetilde{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k) .$$
$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \widetilde{x}(k) + \begin{bmatrix} 0 & 1 \end{bmatrix} u(k)$$

Are these representations equivalent? Are they zero-input equivalent?